Problem Set 4 Thursday June 19, 2003

Problem 1:

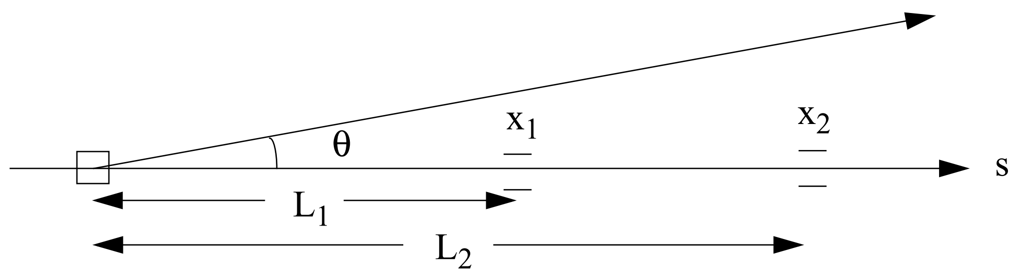

Figure for problem1

1a) The figure shows a simple transport line consisting of a corrector followed by two bpms. The corrector is separated from the bpms by drift spaces L_1 and L_2. Determine the response matrix for this corrector/bpm configuration.

1b) Determine the pseudoinverse of the response matrix R in 9a.

1c) Derive a formula for the corrector kick angle change that minimizes the displacements at the two bpms.

1d) Determine the SVD of the response matrix $R = U S V^T$. The eigenvalues of RR^T (R^TR) are the squares of the singular values and the normalized eigenvectors of RR^T and R^TR are the columns of U and V respectively.

1e) Determine the value of the corrector angle that minimizes the position of the beam at the bpms by minimizing the function:

$$\chi^2 = (x_1 - L_1\theta)^2 + (x_2 - L_2\theta)^2.$$

Problem 2:

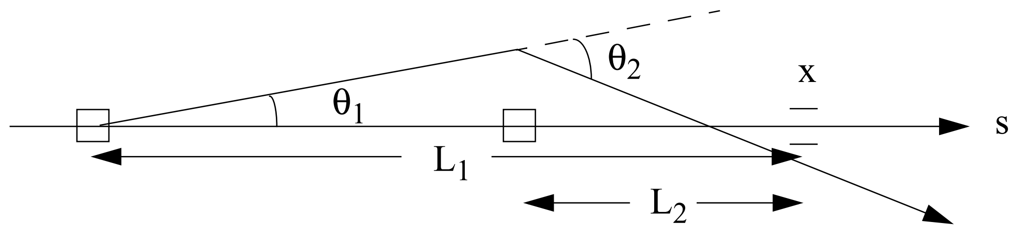

Figure for problem 2

2a) The figure shows a simple transport line consisting of two correctors followed by one bpm. The corrector is separated from the bpms by drift spaces L_1 and L_2. Determine the response matrix for this corrector/bpm configuration.

2b) Determine the pseudoinverse of the response matrix R in 2a.

2c) Determine the SVD of the response matrix $R = U\,S\,V^T$. The eigenvalues of RR^T (R^TR) are the squares of the singular values and the normalized eigenvectors of RR^T and R^TR are the columns of U and V respectively.

2d) Determine the pseudoinverse of the matrix R using the SVD result in 2c.

2e) Use the result of 2d to determine the corrector kick angles that minimize the position at the bpm.

2f) The method of Lagrange multipliers is used to minimize a function subject to a constraint. In this problem, the constraint is that the difference between the bpm position must always equal the sum of the two corrector kicks. The constraint can be expressed by the function:

$$G(\theta_1,\theta_2) = x - L_1\theta_1 - L_2\theta_2 = 0$$

In this problem, the function (χ^2) to minimize with this constraint is the sum of the squares of the corrector kick angles: $\chi^2 = \theta_1^{\,2} + \theta_2^{\,2}$.

Given the function:

$F(\theta_1,\theta_2,\lambda) = \chi^2 + \lambda G(\theta_1,\theta_2)$,

Derive formulas for the corrector kick angles by minimizing $F(\theta_1,\theta_2,\lambda)$ with respect to the angles and λ.

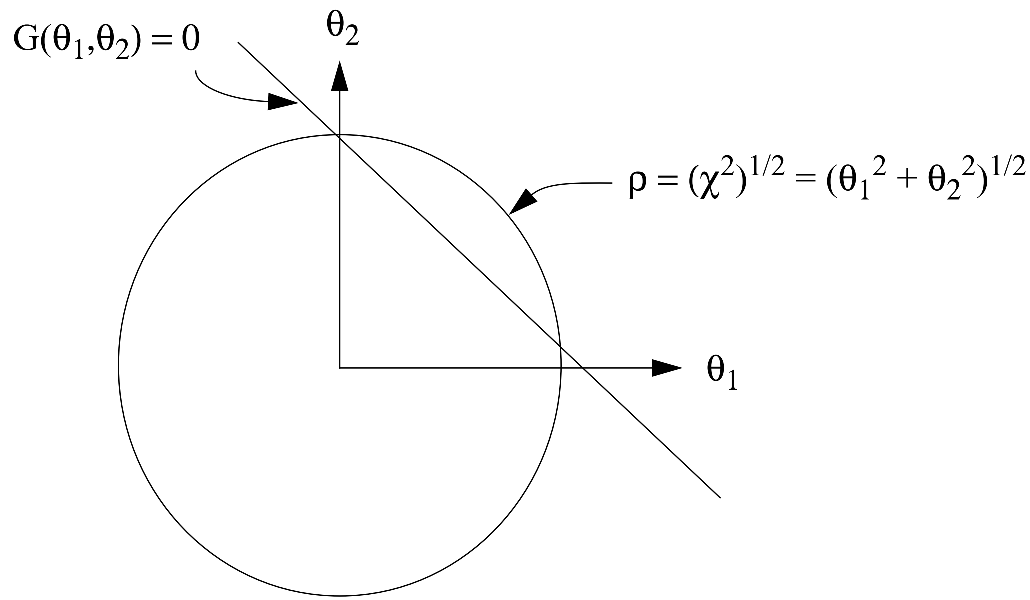

Figure for problem 2g

2g) To illustrate the least squares minimization obtained from the SVD/pseudoinverse method and the method of Lagrange multipliers, consider the constraint function $G(\theta_1,\theta_2)$ and χ^2 in the θ_1, θ_2 plane: Show that the values for θ_1 and θ_2 given by both methods are simply the point where the circle defined by ρ is tangent to the line defined by $G(\theta_1,\theta_2) = 0$.